Example 4: $f(x) = x + \cos(x)$ on $[0, 2\pi]$.

Domain: $[0, 2\pi]$

No "floppy" edge behavior. We will just plug in and plot the endpts!

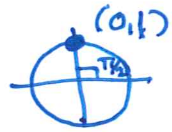
Finding important pts!

$f'(x) = 1 - \sin(x)$

$f'(x)$ is defined everywhere

$f'(x) = 0$ when $\sin(x) = 1$

on $[0, 2\pi]$



this occurs at

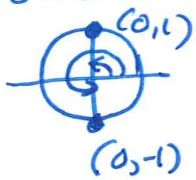
$x = \pi/2$

$f''(x) = -\cos(x)$.

$f''(x)$ is defined everywhere.

$f''(x) = 0$ when $\cos(x) = 0$.

on $[0, 2\pi]$ this occurs at



at $x = \pi/2$

and at $x = \frac{3\pi}{2}$

Increasing/decreasing

f': 0 $\quad + \quad$ $\pi/2$ $\quad + \quad$ 2π

Concave up/concave down

f'': 0 $\quad - \quad$ $\pi/2$ $\quad + \quad$ $\frac{3\pi}{2}$ $\quad - \quad$ 2π

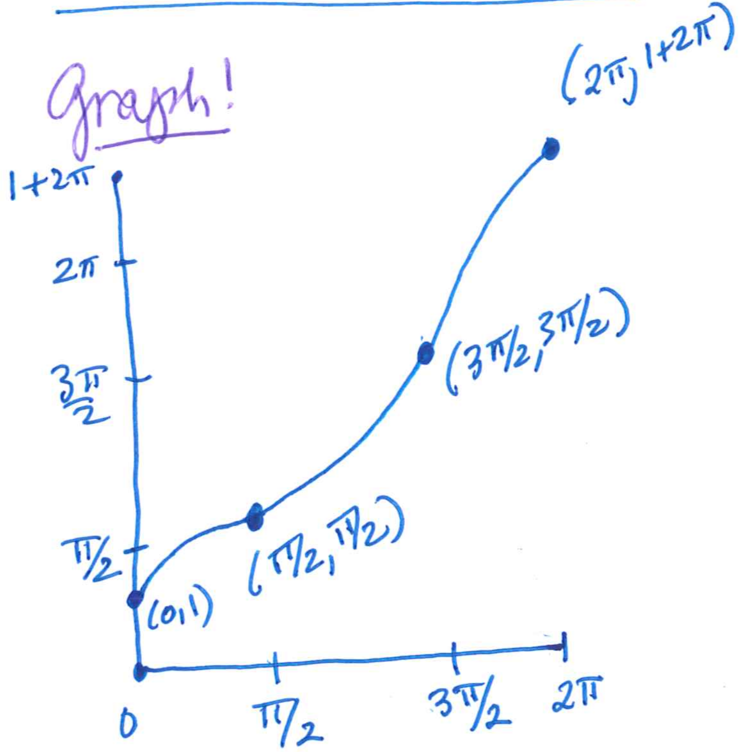
Summary of geometry of the curve

f: 0 $\quad$ incr/$\downarrow$ $\quad$ $\pi/2$ $\quad$ incr/$\uparrow$ $\quad$ $\frac{3\pi}{2}$ $\quad$ incr/$\downarrow$ $\quad$ 2π

Important Points

x	f(x)
0	1
$\frac{\pi}{2}$	$\pi/2$
$\frac{3\pi}{2}$	$3\pi/2$
2π	$1+2\pi$

Graph!



Example 5: $f(x) = 2x^2 \ln(3x)$.

Domain: $x > 0$

"floppy" edge behavior at $x=0$ and toward $+\infty$

$$\lim_{x \to 0^+} 2x^2 \ln(3x) = 0$$

IN MAPLE limit(f(x), x=0, right)

(If you want to know how to calculate this limit yourself, ask me!)

$$\lim_{x \to \infty} 2x^2 \ln(3x) = +\infty$$

Or this one! limit(f(x), x=infinity)

Finding Important Pts.

$f'(x) = 2x\,(2\ln(3x) + 1)$

$f'(x) = 0$ only when

$2\ln(3x) + 1 = 0$

(f' is not defined at $x=0$!)

So $\ln(3x) = -\frac{1}{2}$

$3x = e^{-1/2}$

$x = \frac{1}{3}e^{-1/2} \approx .20$

$f''(x) = 4\ln(3x) + 6$

$f''(x) = 0$ when $\ln(3x) = -\frac{3}{2}$

So $x = \frac{1}{3}e^{-3/2} \approx .07$

Increasing/decreasing

f': 0 — $-$ — $\frac{1}{3}e^{-1/2}$ — $+$ →

Concave up / concave down

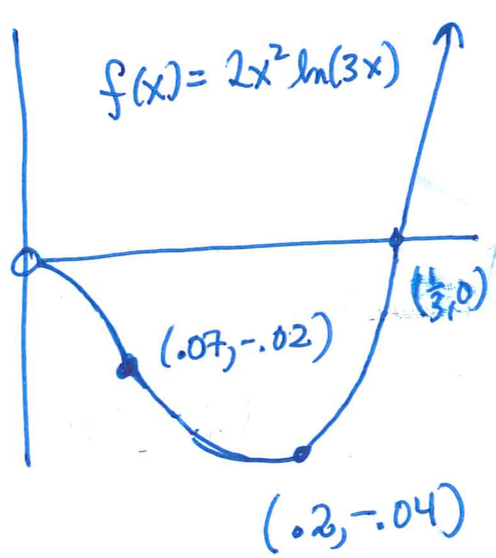
f'': 0 — $-$ — $\frac{1}{3}e^{-3/2}$ — $+$ →

Summary

f: 0 — decr ↓ — $\frac{1}{3}e^{-3/2}$ — decr ↑ — $\frac{1}{3}e^{-1/2}$ — incr ↑ →

x	f(x)
$\frac{1}{3}e^{-1/2}$	$-\frac{1}{9}e^{-1} \approx -.04$
$\frac{1}{3}e^{-3/2}$	$-\frac{1}{3}e^{-3} \approx -.02$

important pts



Graph!